Foreseeable Geometry Improvements from Future Global Navigation Flower Constellation Systems

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Abstract: This paper reports on recent results of simulations carried out for the purpose of evaluating the likely geometric advantages from a future Global Navigation Flower Constellation (GNFC) through comparisons of its performance with that of the conventionally designed constellations of GPS and GALILEO satellite systems. The evaluation of all three satellite systems is performed on the basis of the spatially varying Geometric Dilution of Precision (GDOP) scalar indicator which, at each earth location, reflects at a time instance the contribution of the satellite-receiver geometry which, in turn, affects the resulting accuracy of the navigation solution. The area of interest for the simulations was essentially global, although confined to the zone with latitude bounds -60 deg to +70 deg. The main design concepts and the advantages of the Flower Constellations with respect to the conventional GNSS orbits are discussed. Simulation results are presented in detail which shows that the tested GNFC constellation yields improved navigation performance in terms of GDOP values worldwide, except for a few small areas along the equator and mostly at oceanic regions where, by comparison, the current GPS and GALILEO constellations show slightly better geometric performance.

1. Introduction

The surveying, mapping and spatial information industry has benefited greatly from the use of Global Navigation Satellite Systems (GNSS), involving a multitude of satellites, ground reference station infrastructure, various augmentations and state-of-the-art user equipment which allow determining accurate positions anywhere and anytime around the world. The US Global Positioning System (GPS) is currently the only such fully operational system which is being modernized by adding next generation satellites with more signal frequencies and codes, and with the aim to deliver to the users better accuracy, reliability and availability. Russia also operates GLONASS, its own GNSS, which is being revitalized as well. It is foreseen that fuelling further growth in the coming decade will be next generation GNSS systems, and their respective space augmentations, that are currently being studied for the purpose of improving further the future operational capabilities, especially with regard to observing individual sites or entire regions with optimal global coverage, while minimizing the number of satellites required in space in order to
achieve such increased capabilities. It is within this context that, in this paper, using the concept of the Flower Constellations, we explore a key issue in the design of such constellations: the symmetry in space and/or in time distribution of the satellites which, in turn, affects the user-satellites relative geometry and subsequently the obtainable accuracy of a GNSS solution, and hence the anticipated Levels of Service towards the users.

1.1 Concept and applications of the Flower Constellations

The Flower Constellation (FC) concept was first introduced by Mortari et al. (2003) in an effort to generalize the design of circular inclined orbits having repeatable ground tracks. The basic characteristic of a Flower Constellation is that the orbits of all its satellites are compatible with a rotating reference frame. In other words “when viewed from an arbitrarily defined rotating reference frame, they all follow a single, identical closed-loop reference trajectory” (Wilkins, 2004). If the rotating frame is set to be an Earth Centered Fixed (ECF) frame (i.e. a frame rotating with the earth’s rotation rate), then the satellites will have perfectly repeated ground tracks. Due to the fact that the resulting orbit path of these satellites, as viewed from the rotating reference frame, resembles the shape of a flower petal, such a constellation set is suitably referred to as ‘Flower Constellation’.

In the original Flower Constellation theory, a FC is defined by eight parameters. Three are orbital parameters, which are identical for all the satellites belonging to the same FC: the perigee altitude \(h_p\), the orbit inclination \(i\), and the argument of perigee \(\omega\). The other five are integer parameters playing a special role in defining the relative phasing of the satellites into a sequence of appropriate admissible positions: the number of satellites \(N_s\), the number of sidereal days required in order to repeat the ground track \(N_d\), the number of pedals \(N_p\) which equals the number of revolutions required to complete one period of repetition, and two integer parameters \(F_n\) and \(F_d\) which govern the phasing of the satellites, plus one extra phasing parameter \(F_h\) that was later introduced by Mortari and Wilkins (2008). Over the years, the original Flower Constellation theory has evolved into the so-called 2-D and 3-D lattice theories, as well as the most recent necklace theory (Mortari, 2012), which however are beyond the scope of this paper.

Satellite systems with orbital configurations based on the Flower Constellations concept have been proposed for a large number of applications, ranging from telecommunications and navigation to Earth and Deep Space missions. It has also been shown that FCs permit elliptical orbits, and can support just as well global, regional or spot areas services (e.g., reconnaissance and surveillance satellites, or close formation flying satellites). The fact that FCs are not confined to circular orbits and that they provide an unlimited range of satellite phasing configurations makes them a more efficient, yet complex, choice for a wider range of applications.
than those allowed by conventional constellations, such as the GPS and GALILEO
GNSS systems which have been designed using another satellite constellations de-
sign concept known as the Walker formalism (Ruggieri et al, 2006) which is com-
monly used when seeking symmetrical distributions of satellites in an inertial
frame of reference.

1.2 GDOP Definition

For the purposes of this study, in order to evaluate the GNFC, GPS and GALILEO
constellations under consideration, we are going to use the well-known to GPS us-
ers GDOP metric which is dependent entirely on the geometry of the satellites
within the view of a given location of interest. Using the expressions (1) and (2)
below, GDOP is defined as the ratio of the square root of the trace of the covari-
ance matrix \( C_{\hat{x}} \) for a GPS navigation (instantaneous positioning) solution di-
vided by the accuracy of a single pseudorange measurement, expressed by the User
Equivalent Range Error \( \sigma_{\text{UERE}} \) or the associated standard deviation of the actual
measurements. Usually, for convenience and without compromising the resulting
accuracy of the relevant computations, it is assumed that we are dealing with inde-
pendent and identically distributed random range errors, i.e. the non-diagonal ele-
ments of \( C_{\hat{x}} \) are zero (Yarlagadda et al, 2000; Langley, 1999).

\[
C_{\hat{x}} = \begin{bmatrix}
\sigma_x^2 & \sigma_{xy} & \sigma_{xz} & \sigma_{xt} \\
\sigma_{yx} & \sigma_y^2 & \sigma_{yz} & \sigma_{yt} \\
\sigma_{zx} & \sigma_{zy} & \sigma_z^2 & \sigma_{zt} \\
\sigma_{tx} & \sigma_{ty} & \sigma_{tz} & \sigma_t^2 \\
\end{bmatrix}
\]

\[GDOP = \frac{\sqrt{\text{trace}(C_{\hat{x}})}}{\sigma_{\text{UERE}}} = \sqrt{\frac{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 + \sigma_t^2}{\sigma_{\text{UERE}}}}\]  

Small GDOP values indicate good satellite geometry which results in an instanta-
neous position having small uncertainty. When the GDOP values increase from
epoch to epoch, it is indicative that the satellite geometry gets poorer and conse-
quently the position fix uncertainty gets worse (Μπούσιας-Αλεξάκης, 2013).

2. Simulation considerations

For the current study we conducted various simulations with the aim to compare
the geometric performance of a typical Flower Constellation for global navigation
purposes, with the performance of the current GPS constellation and the currently
implemented constellation of the GALILEO system satellites. The defining pa-
rameters and other key technical characteristics for these constellations, as used in the simulations, are briefly discussed below.

2.1 Global Navigation Flower Constellation

Park et al (2004) have suggested a FC for global navigation purposes, which has been named Global Navigation Flower Constellation (GNFC). This particular orbit configuration design essentially consists of two separate FCs, each containing 15 satellites. The parameters defining each FC are presented in Table I.

<table>
<thead>
<tr>
<th>Table I: Flower Constellation Parameters (Park et al, 2004)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FC Parameters</strong></td>
</tr>
<tr>
<td>N_p</td>
</tr>
<tr>
<td>N_d</td>
</tr>
<tr>
<td>N_s</td>
</tr>
<tr>
<td>(\omega)</td>
</tr>
<tr>
<td>(i)</td>
</tr>
<tr>
<td>h_p</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table II: GNFC Parameters: (\Omega) &amp; (M) in degrees (Park et al, 2004)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sat. #</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>13</td>
</tr>
<tr>
<td>15</td>
</tr>
</tbody>
</table>

By using a symmetric phasing scheme (i.e. uniformly distributed satellites in the constellation’s relative path) and assuming that the mean anomaly at the initial time \(M_0\) and the right ascension of the ascending node (RAAN) of the first satellite are zero, the data for all 15 satellites can be defined by the parameters presented in Table II. The two FCs have identical orbital elements and the only difference between them is that the argument of perigee is \(\omega = 90\) deg for one of them and \(\omega = 180\) deg for the other.

2.2 GPS Constellation

The GPS system was originally designed as a Walker 24/3/1 constellation, where
24 refers to the total number of satellites, 3 refers to the number of the orbital planes and 1 refers to the phasing parameter that defines the relative spacing between satellites in adjacent orbital planes. Although this pattern provided worldwide continuous coverage by at least 4 satellites at all times, it proved too sensitive to likely satellite failures. This eventually led to the system’s re-designed constellation of 24 satellites in 6 equally spaced orbit planes, with nominal values for the semi-major axis, the orbital planes’ inclination, and the eccentricity set to: 

\[ a = 26,559.7 \text{ km}, \quad i = 55 \text{ deg} \quad \text{and} \quad e = 0 \], respectively (DoD, 2008).

In 2011 this nominal constellation was modified so as to include three more slots in the core constellation and has since evolved into the so-called 24-expandable baseline GPS constellation. In addition, spare satellites have been placed in orbit which are fully operational, but do not occupy a slot in the baseline constellation. In accordance with today’s configuration, in our simulation scenarios we specified that at start time the GPS constellation consisted of 31 satellites.

2.3 GALILEO Constellation

Similarly, the GALILEO Constellation, in our simulations, was also designed a 27/3/1 Walker Constellation with an orbital inclination \( i = 56 \text{ deg} \) and an orbital radius of 23,222 km. A spare operational satellite was also provided for each of the three orbital planes.

3. Simulation setups

3.1 Creating the scenarios- Inserting the Constellations

In order to carry out our simulations, we used the Satellite Tool Kit (STK 9) by Analytical Graphics Inc. (AGI). We created three scenarios, one for each GNSS system constellation. The time period for each scenario was set to ten days, which is the same as the subtrack repetition period for the GALILEO satellites and also the longest subtrack repetition period amongst all three GNSS constellations.

The next step was to include in the simulations all three constellations in each of the three respective scenarios. We simulated the orbits of the 31 GPS satellites by using the SGP4 Simplified Perturbations Propagator and the Two Line Elements (TLE) orbital sets provided by the STK’s satellite database. During the simulation period covered by each scenario and for the orbit propagation of each satellite, the STK would switch automatically between appropriate TLE sets in its database, so that orbital elements closer to the epochs of the simulated observations were always used. Similarly, we used STK’s Walker Tool and inserted, as a 30/3/1 Walker constellation, the GALILEO satellites having an inclination \( i = 56 \text{ deg} \) and an orbital altitude of 23,222 km. Finally, the GNFC constellation was included in the
simulation by inserting all the satellites of the two sub-FCs specified in Tables I and II. In both cases, for the propagation of each GNFC orbit, we used the J4 (second-order) propagator which accounts for secular variations in the orbit elements due to Earth’s oblateness, thus avoiding any likely perturbations which would otherwise slowly destroy the in-orbit compatibility of the GNFC satellites.

3.2 Definition and sampling of the Area of Interest

The area of interest for the current simulations was defined as the zone within the latitudes -60 deg to +70 deg, as this is the inhabited part of the globe where most human activities are taking place. A grid of 4212 points, separated in both latitude and longitude by a spacing of 3 deg at the equator, was created in order to analyze the time-varying GDOP values throughout the area of interest.

3.3 Computation of the Static GDOP values

The GDOP value for any point in space changes over time. Therefore, in order to evaluate and compare the quality of coverage that each constellation provides at each grid point, we have to use a representative static-like GDOP value, which covers the duration of each simulation scenario. Specifically, at each grid point over the duration of a scenario, the following variants of GDOP values were used: the average GDOP value (\(aGDOP\)); the maximum GDOP value (\(maxGDOP\)); and the upper bound limit (\(upbGDOP\)) that the GDOP values did not exceed for 90 percent of the times. Each of these static-like GDOP values for each grid point was computed from the time-varying values of GDOP, using a sampling time step of 300 sec. At each epoch, the corresponding GDOP value was always calculated using all visible satellites at that epoch that had an elevation angle greater than 10 deg. In addition, the minimum number of available satellites observed over the entire duration of a scenario was calculated and stored for each grid point.

3.4 Comparison Process

In order to compare the geometric performance of the GNFC to the corresponding performance for each of the GPS and GALILEO constellations, we first calculated the following differences between the respective static-like GDOP values produced by different satellite constellations at the \(i^{th}\) grid point:

\[
\Delta GDOP_i' = GDOP_i^{GPS} - GDOP_i^{GNFC}
\]

\[
\Delta GDOP_i'' = GDOP_i^{GALILEO} - GDOP_i^{GNFC}.
\]

In those expressions, for \(i = 1...4212\), \(GDOP_i^{GNFC}\) denotes the static-like GDOP value (e.g. \(aGDOP\), \(maxGDOP\) or \(upbGDOP\)) which occurred for the GNFC constellation at the \(i^{th}\) grid point, and \(GDOP_i^{GPS}\) and \(GDOP_i^{GALILEO}\) are defined in the same way for GPS and GALILEO. In that way, we obtained respectively: the
$a\Delta GDOP_i$ and $a\Delta GDOP_i$" indicators, when we used in (3) and (4), the average GDOP value at each point; the $max\Delta GDOP_i$ and $max\Delta GDOP_i$" indicators, when we used the maximum GDOP value; and the $upb\Delta GDOP_i$ and $upb\Delta GDOP_i$" indicators, when we used the upper bound limit that the GDOP values did not exceed for 90% of the times over the duration of a scenario. It should be noted that the term ‘90%below’, is used for simplicity, in the tables and the discussions that follow, in order to denote the $upb\Delta GDOP_i$ and $upb\Delta GDOP_i$" indicators.

The calculated differences $\Delta GDOP_i$ and $\Delta GDOP_i$" are not transparent in understanding the amount of improvement or deterioration in the quality of the time-varying geometric coverage offered by each constellation. A more intuitive measure of performance is given by the expressions of relative percentage rate of change of the respective static-like GDOP values (e.g. $aGDOP$, $maxGDOP$ or $upbGDOP$), between GNFC and each of the GPS or GALILEO constellations:

\[
%\Delta GDOP_i' = \left[ \frac{(GDOP_i^{GPS} - GDOP_i^{GNFC})}{GDOP_i^{GPS}} \right] 100\%
\]  
\[
%\Delta GDOP_i'' = \left[ \frac{(GDOP_i^{GALILEO} - GDOP_i^{GNFC})}{GDOP_i^{GALILEO}} \right] 100\%
\]

It should be mentioned that all the necessary computations above, as well as the plotting for these quality indicators and their subsequent analyses were carried out by using various MATLAB scripts that were developed and worked within STK in a transparent way.

4. Results

4.1 Results for each Constellation independently

A simple statistical analysis has been applied in order to provide an overview of the geometric performance exhibited by each constellation. In Table III, the first three columns refer to the static-like GDOP values that were calculated for each grid point (average, maximum or 90%below), while the rows refer, for each GNSS system, to the typical statistics (average, minimum and maximum) obtained from the GDOP values of all 4212 grid points. For example, the element showing the value 1.71 in the first row for the GNFC system refers to the minimum observed value among the averaged GDOP values obtained at the 4212 points of the grid, from the scenario simulating the G/glyph817FC constellation. In addition to various static-like GDOP values, also shown are the statistics of the minimum number of visible satellites per grid point for each constellation, for the duration of each simulation scenario. From Table III, it is clear that for almost every indicator, the GNFC provides better results, in terms of the ‘average’, ‘maximum’ and ‘90%below’ GDOP values, than those of the current GPS and the upcoming GALILEO systems. In order to inter-compare further the results between the GNFC and each of the GPS and GALILEO constellations, we have also computed the relative percentage differ-
Table III: Statistical GDOP values for GNFC, GPS and GALILEO constellations.

<table>
<thead>
<tr>
<th>Grid Stats</th>
<th>Average</th>
<th>Maximum</th>
<th>90% below</th>
<th>Min. Number of visible satellites</th>
</tr>
</thead>
<tbody>
<tr>
<td>GNFC</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>1.71</td>
<td>1.85</td>
<td>1.79</td>
<td>6.00</td>
</tr>
<tr>
<td>Average</td>
<td>2.09</td>
<td>2.62</td>
<td>2.47</td>
<td>7.80</td>
</tr>
<tr>
<td>Maximum</td>
<td>2.79</td>
<td>4.45</td>
<td>3.36</td>
<td>10.00</td>
</tr>
<tr>
<td>GPS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>1.88</td>
<td>2.44</td>
<td>2.11</td>
<td>4.00</td>
</tr>
<tr>
<td>Average</td>
<td>2.14</td>
<td>4.66</td>
<td>2.71</td>
<td>6.20</td>
</tr>
<tr>
<td>Maximum</td>
<td>2.41</td>
<td>18.43</td>
<td>3.64</td>
<td>8.00</td>
</tr>
<tr>
<td>GALILEO</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>1.96</td>
<td>2.27</td>
<td>2.24</td>
<td>6.00</td>
</tr>
<tr>
<td>Average</td>
<td>2.15</td>
<td>3.46</td>
<td>2.63</td>
<td>6.96</td>
</tr>
<tr>
<td>Maximum</td>
<td>2.39</td>
<td>4.30</td>
<td>4.15</td>
<td>9.00</td>
</tr>
</tbody>
</table>

Table IV: Comparison between GNFC and GPS (indicator %ΔGDOP,') as well as between GNFC and GALILEO (indicator %ΔGDOP,")

<table>
<thead>
<tr>
<th>Grid Stats</th>
<th>Average</th>
<th>Maximum</th>
<th>90% Below</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comparing GNFC to GPS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>9.1</td>
<td>24.2</td>
<td>15.2</td>
</tr>
<tr>
<td>Average</td>
<td>2.5</td>
<td>43.7</td>
<td>8.9</td>
</tr>
<tr>
<td>Maximum</td>
<td>-15.6</td>
<td>75.8</td>
<td>7.8</td>
</tr>
<tr>
<td>Comparing GNFC to GALILEO</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>12.88</td>
<td>18.78</td>
<td>20.32</td>
</tr>
<tr>
<td>Average</td>
<td>2.69</td>
<td>24.14</td>
<td>6.09</td>
</tr>
<tr>
<td>Maximum</td>
<td>-16.47</td>
<td>-3.46</td>
<td>19.23</td>
</tr>
</tbody>
</table>

ences expressed by (5) and (6) and the corresponding results are presented in the following section.

4.2 Comparing GNFC to GPS and GALILEO constellations

The statistical measures %ΔGDOP,′ and %ΔGDOP," resulting from the comparison between the GNFC and the GPS (and respectively the GALILEO) constellation are shown in Table IV. Positive numbers indicate GNFC improvements in GDOP values and, therefore, better satellite geometry and smaller position uncertainty. Evidently, for the majority of the results, the use of the GNFC leads to improved GDOP values. Only in the maximum value occurring amongst the averages of
GDOP indicators at all grid points, the GNFC presents worse results than GPS and GALILEO.

Figures 1 and 2 depict the percentile differences, i.e. the values computed using expression (5), between GPS and GNFC for the “90%below” (i.e., the $\text{upb} \Delta \text{GDOP}_i$) and respectively the “average” (i.e., the $\text{a} \Delta \text{GDOP}_i$) GDOP values.

**Figure 1:** Percentile Differences between GPS and GNFC for the "90% Below" GDOP values (i.e. equ. 5, $\text{upb} \Delta \text{GDOP}_i$).

**Figure 2:** Percentile Differences between GPS and GNFC for the “Average” GDOP values (i.e. equ. 5, $\text{a} \Delta \text{GDOP}_i$).
Similarly, Figures 3 and 4 depict the percentile differences, i.e. the values computed using expression (6), between GALILEO and GNFC for the “90%below” (i.e., the $upbΔGDOP_i$) and respectively the “average” (i.e., the $aΔGDOP_i$) GDOP values. In all four figures, the tones of grey which correspond to positive values illustrate areas where the GNFC presents better results than the GPS or GALILEO constellation. It is also noticeable that, in all cases, there are some “spotty” areas

**Figure 3:** Percentile Difference between GALILEO and GNFC for the “90% Below” GDOP values (i.e. equ. 6, $upbΔGDOP_i$).

**Figure 4:** Percentile Difference between GALILEO and GNFC for the “Average” GDOP values (i.e. equ. 6, $aΔGDOP_i$).
along the equator (indicated by the black color) where GPS (in figures 1 and 2) and respectively GALILEO (in figures 3 and 4) both exhibit better results.

5. Conclusion

In this study we have considered the problem of designing future GNSS systems and employed the Flower Constellations concept to evaluate an alternative GNFC design that, as supported by the results of our simulations, generally provides improved geometric performance, except in a few “spotty” small areas along the equator, where the current GPS and the upcoming GALILEO constellations exhibit slightly better geometric results. This is a characteristic of such FC designs that cannot be avoided. However, using a slightly more judicious choice of orbital parameters of the FC configuration, such areas can be placed in the least required regions (e.g. in oceanic areas).

One of the main complications that we were able to overcome in the present study was that the metrics by which a constellation is evaluated must be computed over time. Given the dynamic nature of this problem, this was accomplished by accounting for the orbital perturbations due to the Earth’s oblateness through the propagation of all satellites in a given constellation for some time of interest (the period for repeating ground-tracks), in order to compute the metric of interest in an extensive grid of points. Overall, this study has shown that future GNSS constellations may benefit from the above demonstrated results and other similar simulations planned as an extension of this study, in particular with regard to studying (and accounting for) the likely impacts of geometric performance in the Levels of Services to be provided by the future GNSS constellations.

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